Radial Symmetry Detection and Shape Characterization with the Multiscale Area Projection Transform

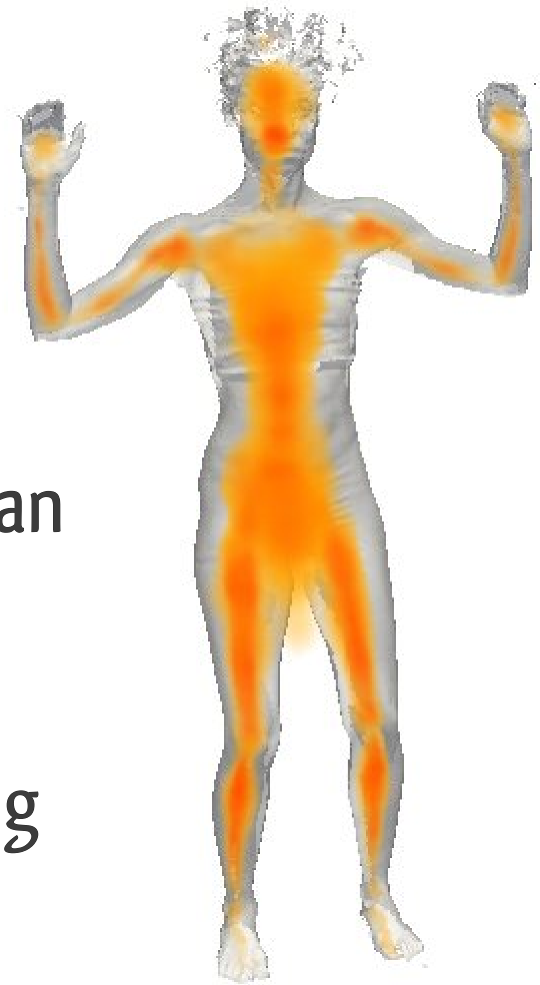
Andrea Giachetti
Department of Computer Science, University of Verona, Italy

andrea.giachetti@univr.it

Christian Lovato
Department of Neurological, Neuropsychological, Morphological and Movement Sciences, University of Verona, Italy

Motivation

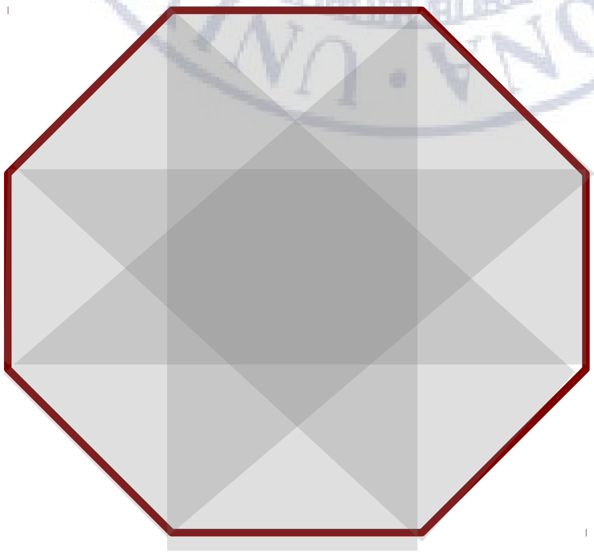
- Initial goal: obtain robust descriptors for "human" salient points/lines
  - Application: anthropometric analysis
- Curve skeletons and other features not robust against noise/holes
- Search for cylindrical/spherical symmetry can be a solution
- Geometric derivation of a radial symmetry based transform revealed several interesting properties…



→ Multiscale Area Projection Transform

Basic idea

- Creating a volume density higher near sphere and cylindrical structure centers
- Not directly exploited in shape analysis methods
- Widely used in the image processing domain



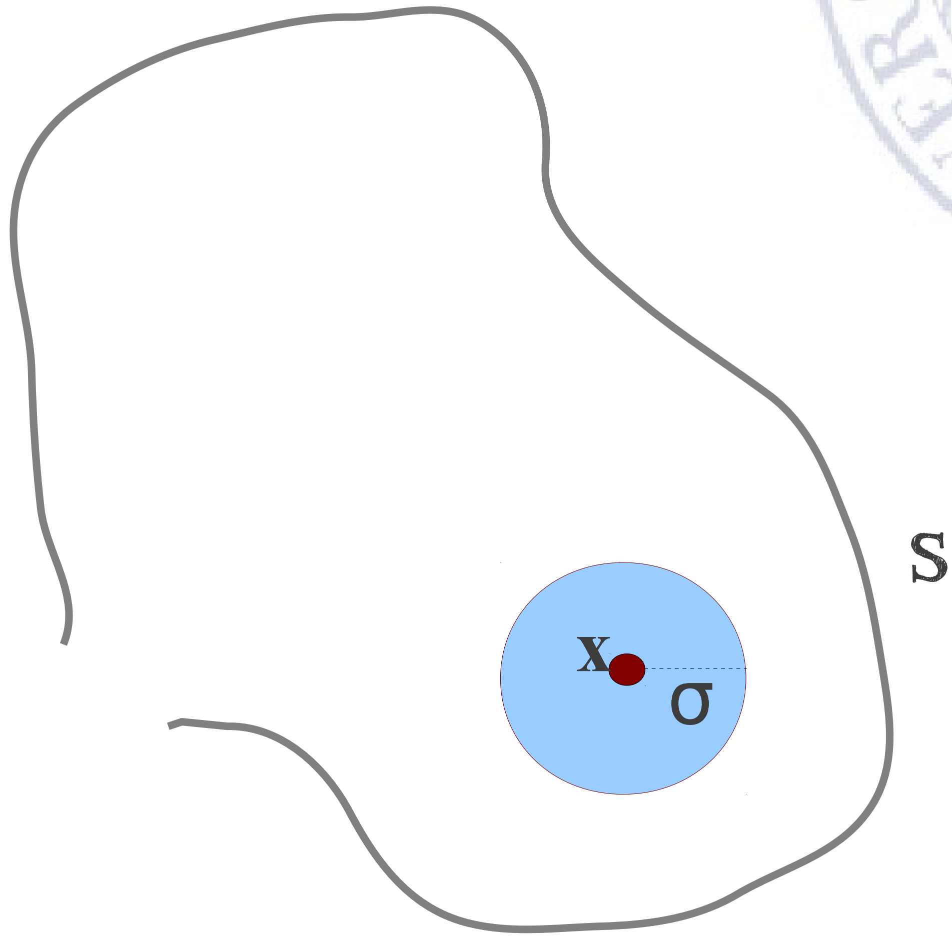
Related work (geometry processing)

- Salient points related to curvature
- Medial axes and curve skeletons extracted as subsets of medial axes
- Segmentation of tubular parts (Mortara et al. '04)
- Skeletonization related to curvature or radial symmetry, e.g.
  - Shape Diameter Function (Shapira et al, 08), robust curve skeletons (e.g. Tagliasacchi et al '09, Livesu et al'12),
- Symmetry maps, e.g. Planar-reflective transform (Podolak et al.'06), etc.
- Use of symmetry for shape retrieval (Kazhdan et al.'04)

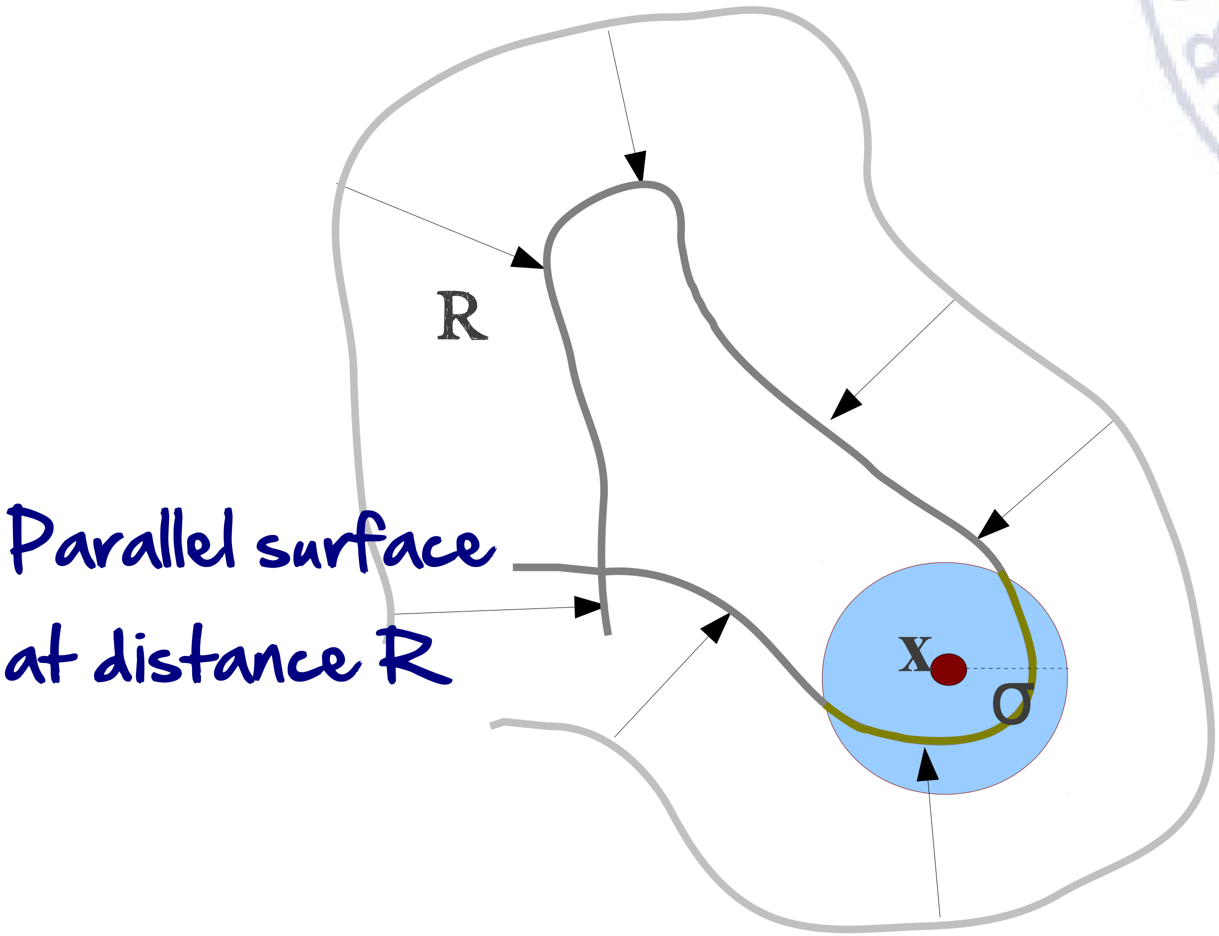
Related work (image processing)

- Circle detection methods, e.g.Reisfeld et al. '95, Sela and Levine '96
- **Fast Radial Symmetry** (Loy & Zelinsky, 03): create a symmetry map projecting edge at increasing distances and counting projected components (creating only a joint map)
- Similar to what we did, but
  - The original geometric framework makes the method more general not depending on image grid, discretization, data structures
  - Exact area weighting is a relevant factor, scales separated
  - Effective application to global shape characterization and **shape retrieval**
  - Links with medial axes and curve skeletons

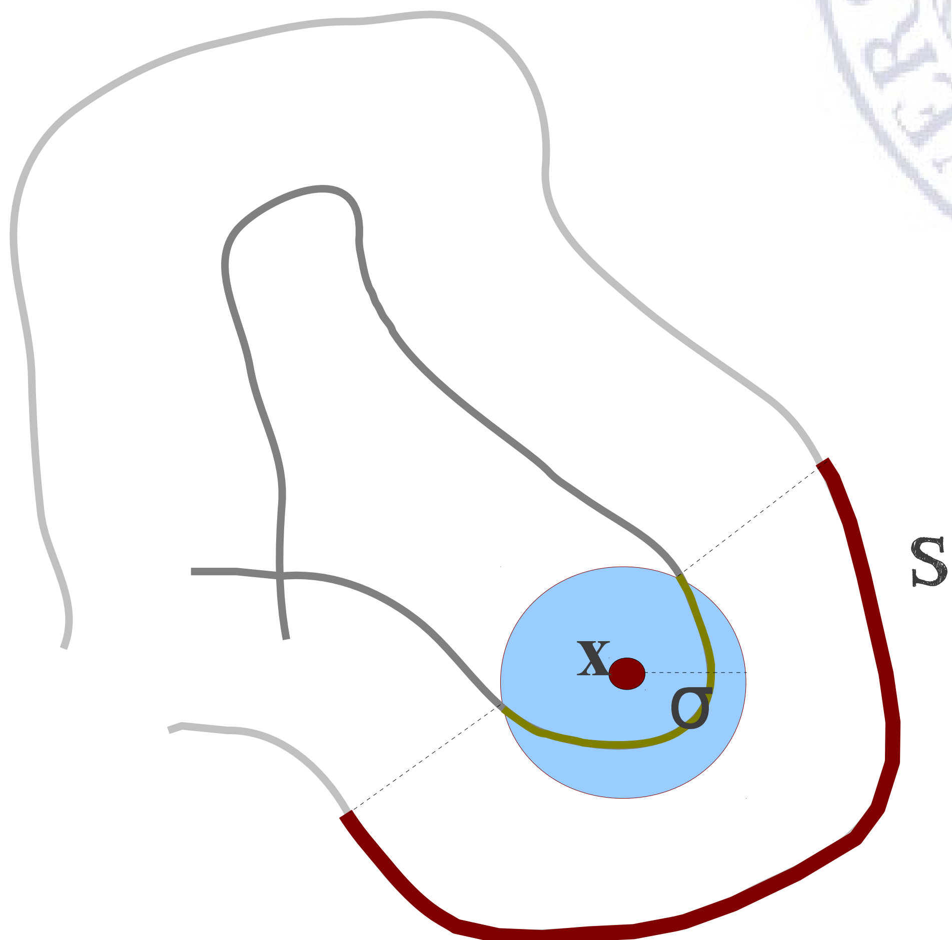
Area projection transform



Area projection transform



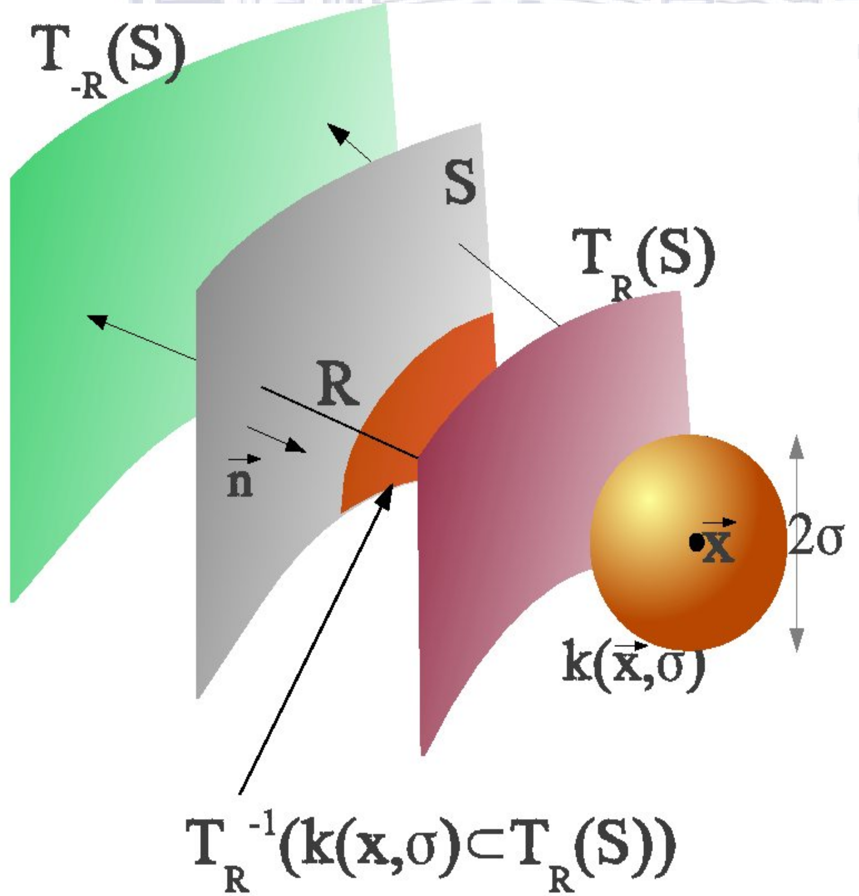
Area projection transform



$$APT(\mathbf{x},R,\sigma) = Area(S)$$

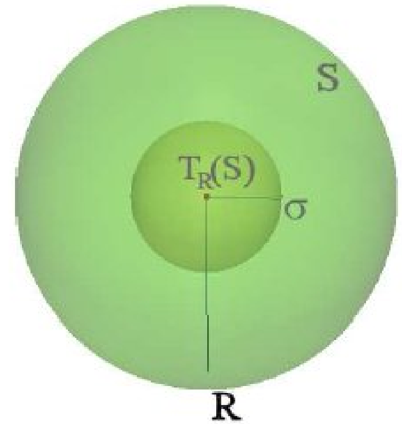
Area projection transform

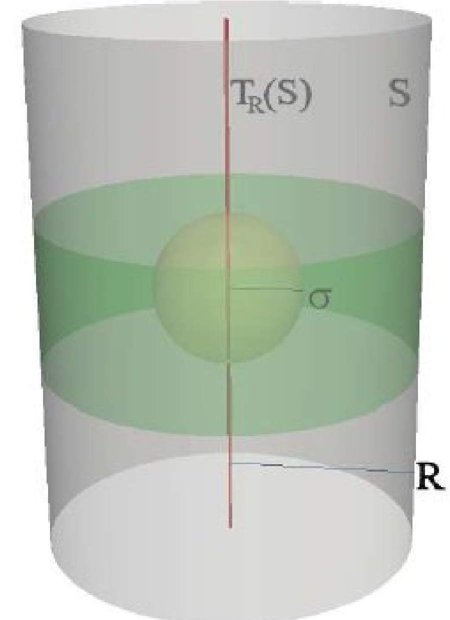
- Consider "internal" parallel surface (Wintner '52) at distance R or both
- Take the part included in a sphere with radius s around **x**
- Compute the area of the corresponding original surface



$$APT(\vec{x},S,R,\sigma) = Area(T_R^{-1}(k_\sigma(\vec{x}) \subset T_R(S,\vec{n})))$$

Properties





- Spheres and cylinders create maxima with known values ($4/3\ \pi R^3$, $4\ \pi\ \sigma$)

- We can compute area density and compute its limit for $\sigma \rightarrow 0$

$$\rho_{AP}(\vec{x}, S, R) = \lim_{\sigma \to 0} \frac{APT(\vec{x}, S, R, \sigma)}{(4/3)\pi\sigma^3}$$

-It is possible to define alternative APT computing kernel density with different kernels

-In the limit density is nonzero only in centers of exact spherical symmetry

-Kernel size determine the amount of “approximate symmetry” detected

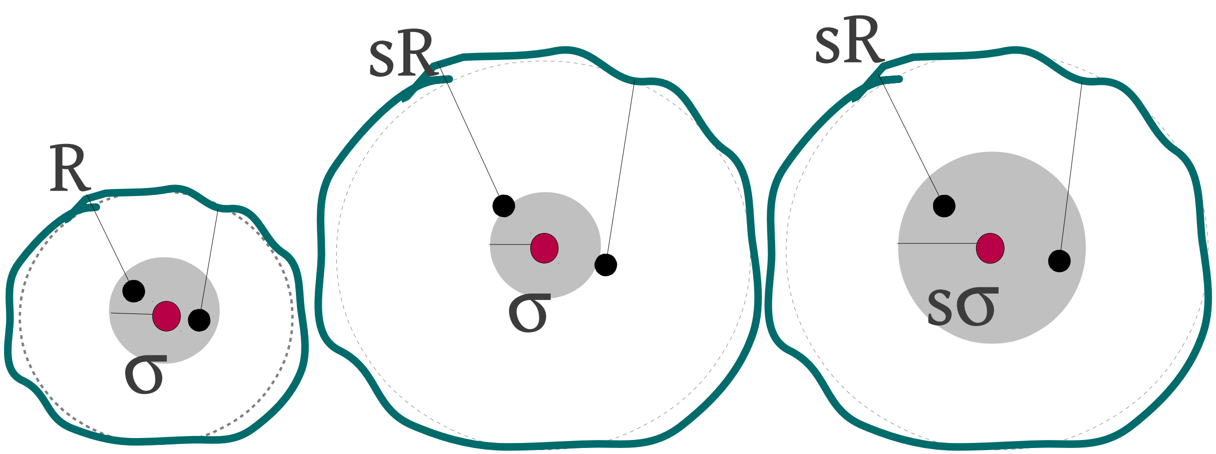
Multiscale APT

- Goal: similarly characterize symmetries at different scales
- Idea: compute APT for varying R in a defined range, but with a scale related normalization in order to:
  - Obtain equal maxima related to spheres and cylinders
  - Scaled shapes should create similarly scaled values
- Definition:

$$\mathrm{MAPT}(x,y,z,R,S) = \alpha(R)\ \mathrm{APT}(x,y,z,R,\sigma(R),S)$$
$$\alpha(R) = 1/(4\pi R^2)$$
$$\sigma(R) = c \cdot R$$
$$(0 < c < 1)$$

Maxima; 1 (spheres)

c (cylinders)



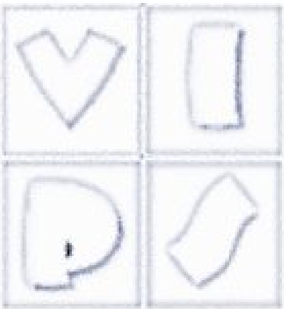
Joint-Multiscale APT

- To save memory it is possible to store only maps with maximal values across scales and related scale

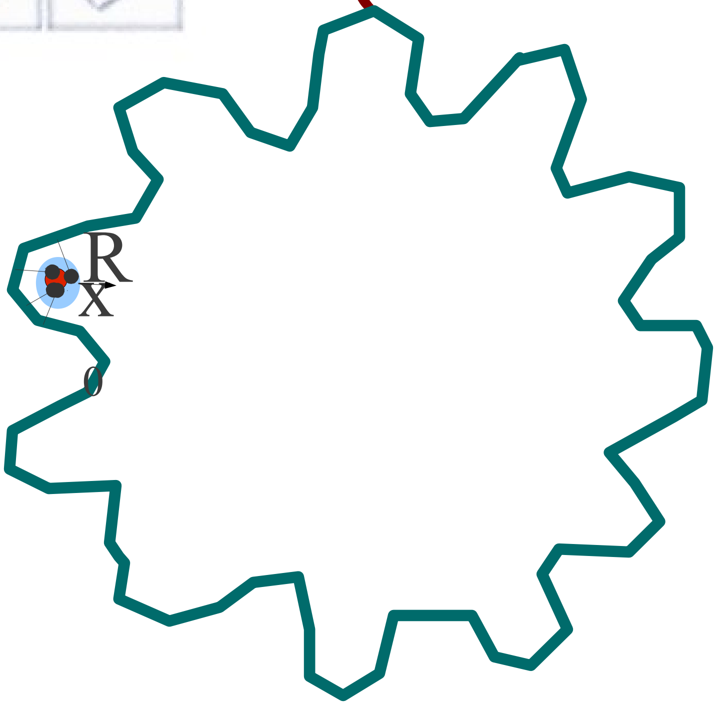
$$JMAPT(x,y,z,S) = \max_{R} APT(x,y,z,R,S)$$

$$SMAPT(x,y,z,S) = \operatorname{argmax}_{R}(APT(x,y,z,R,S))$$

- Can be useful as well for salient points/lines detection



Handling noise
(structures with lower radius)



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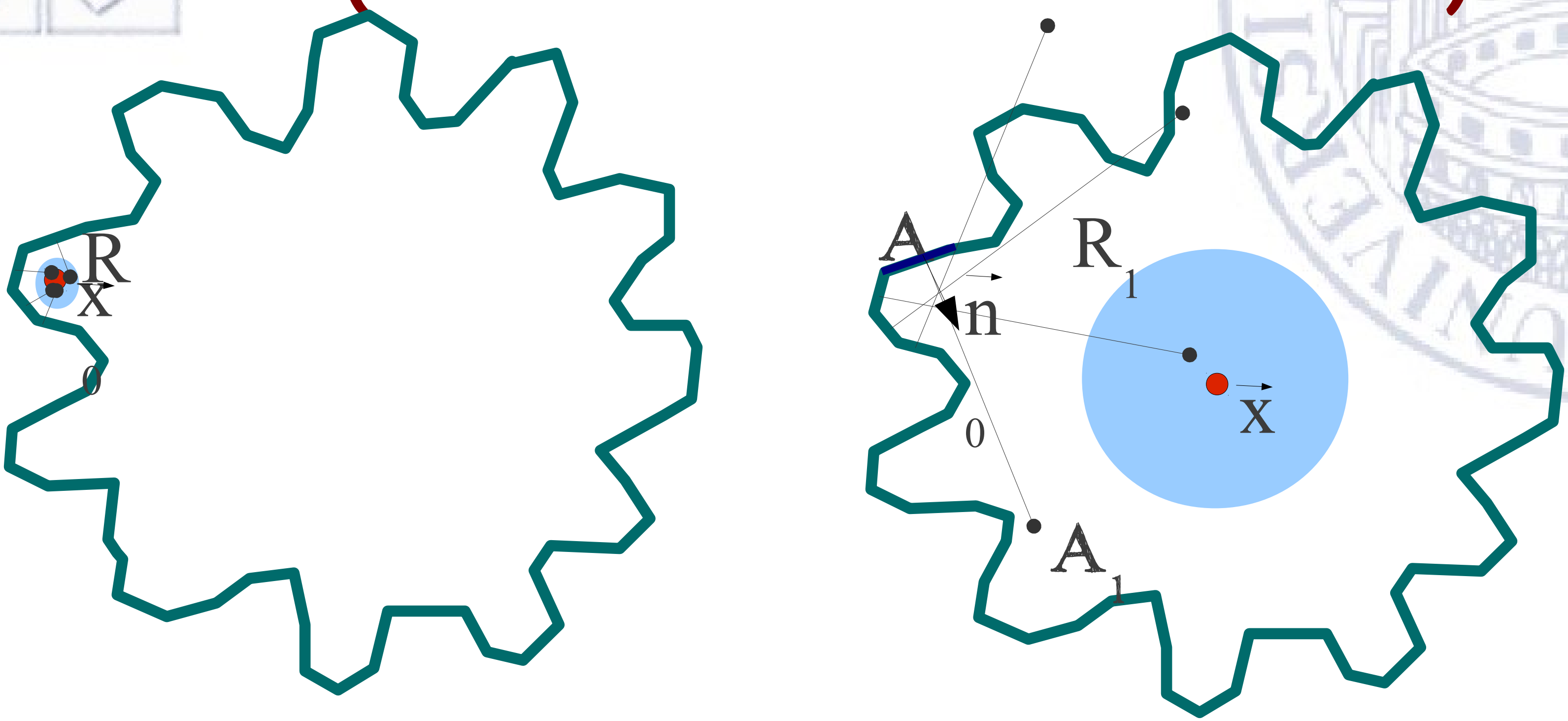

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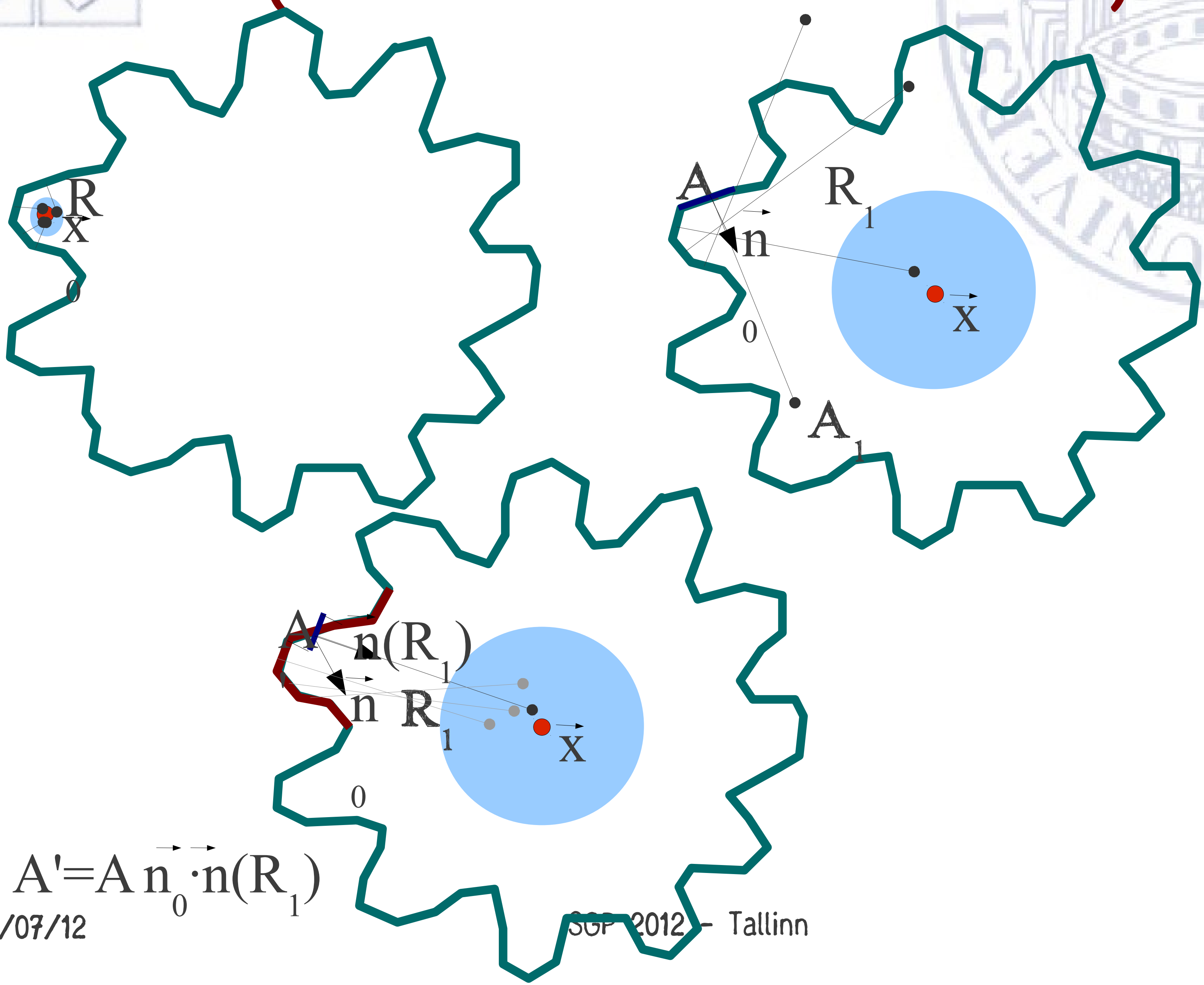

$$A' = A\,\vec{n}_0 \cdot \vec{n}(R_1)$$

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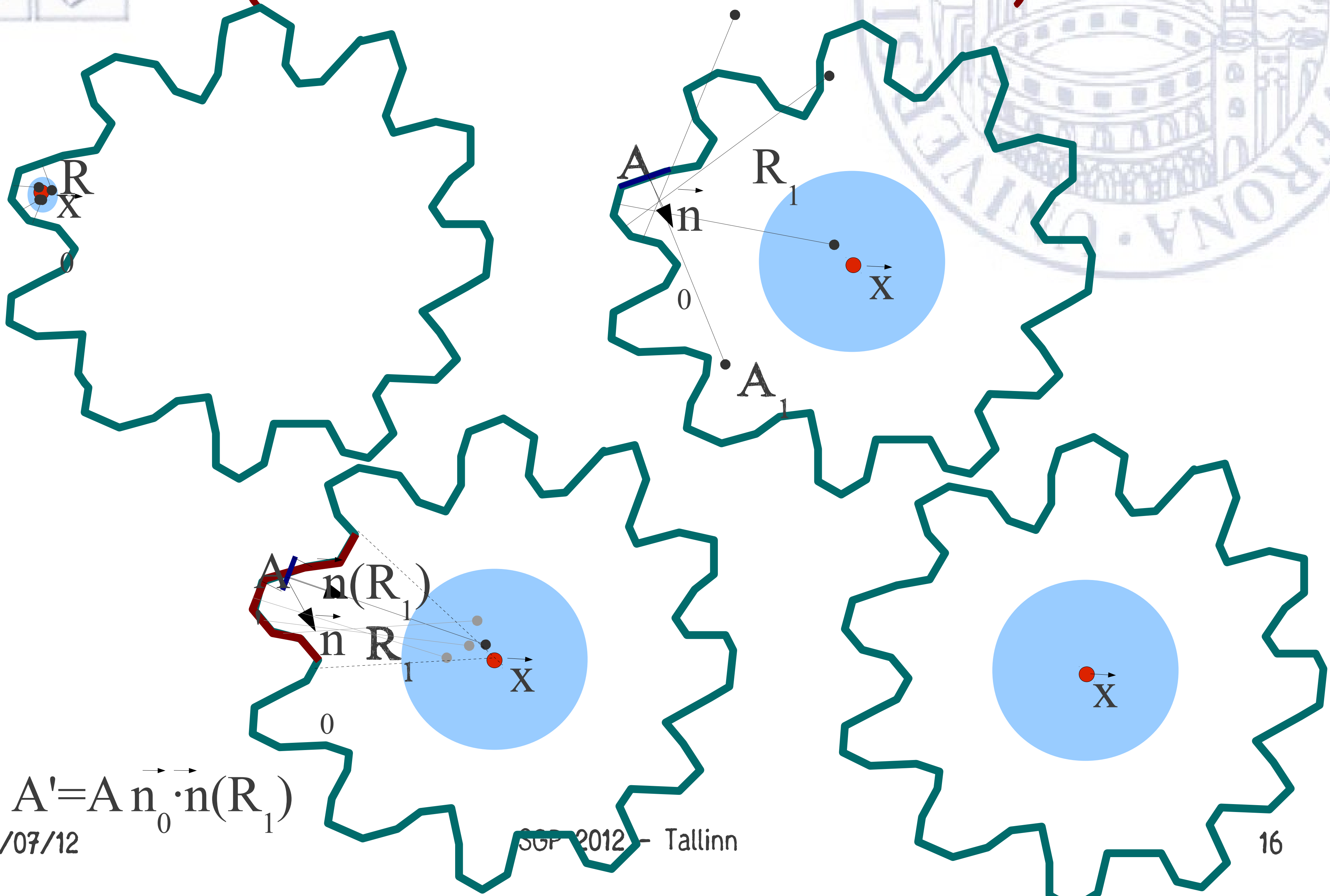

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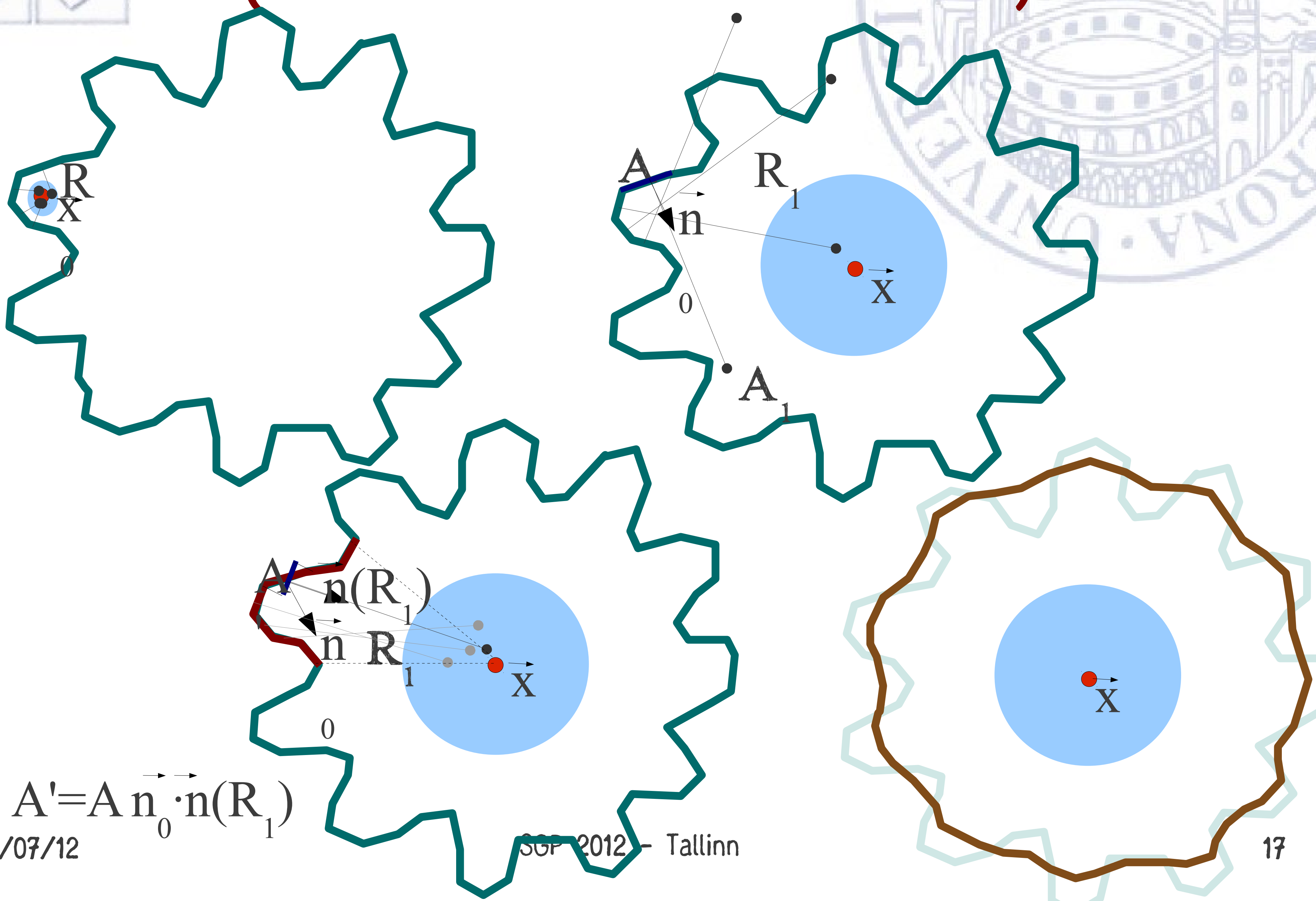

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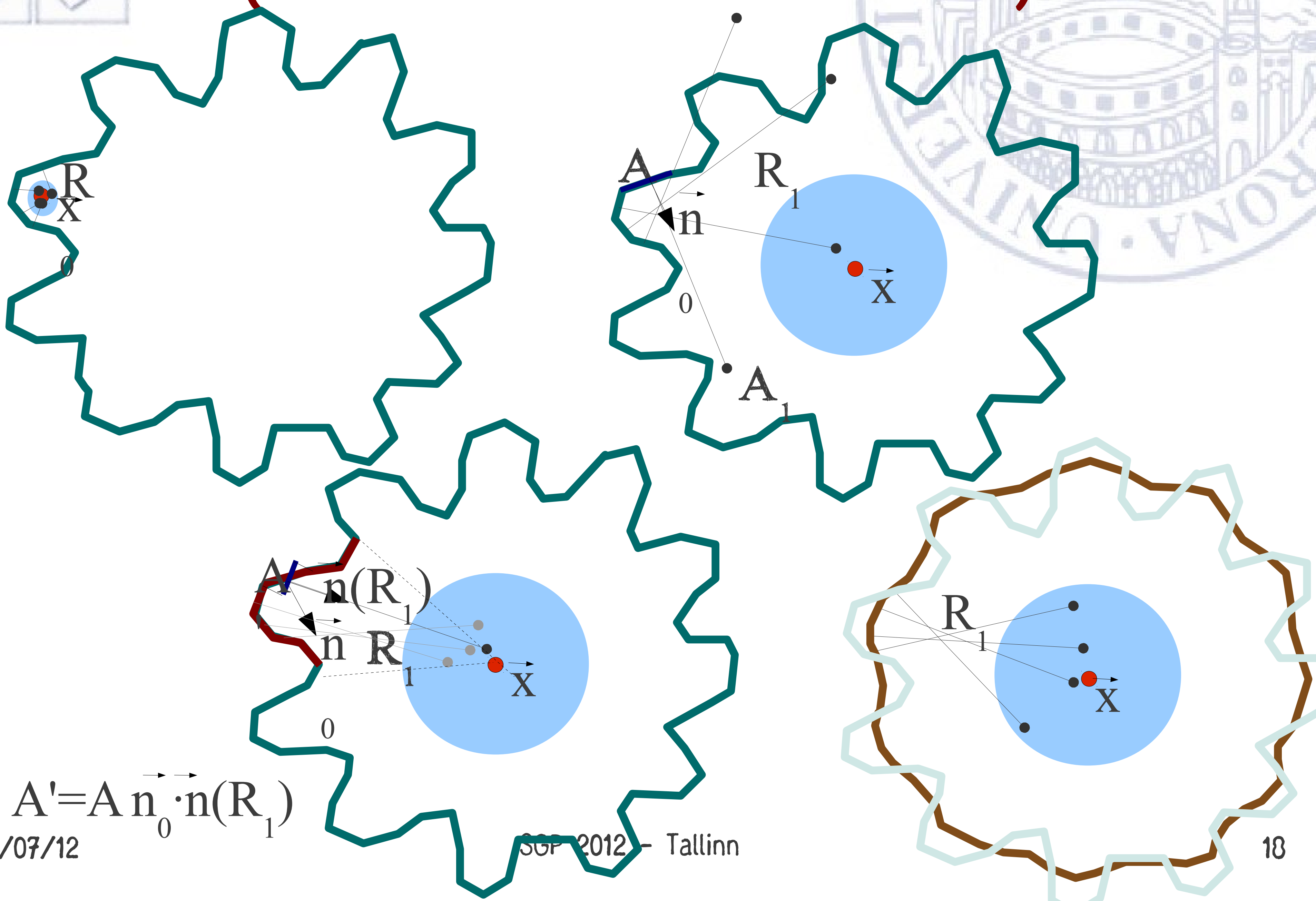

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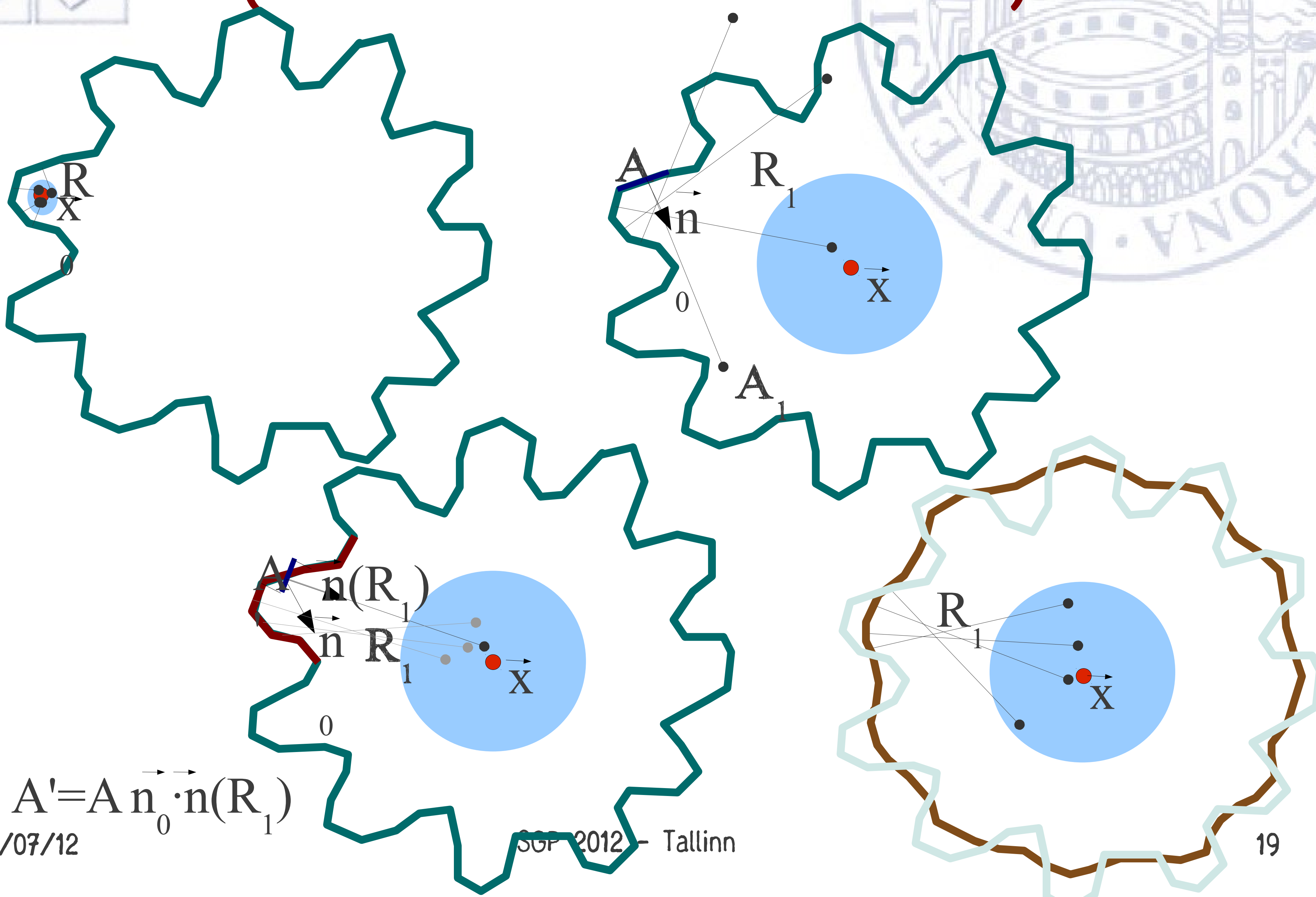
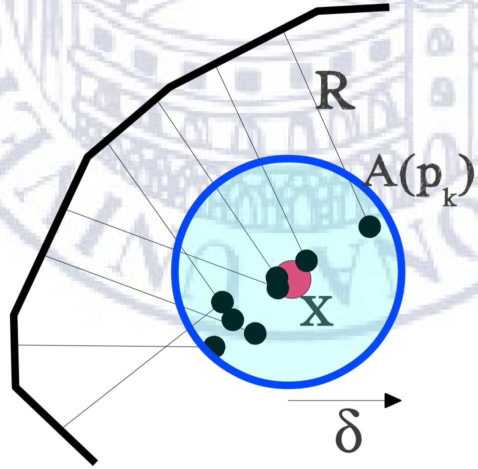
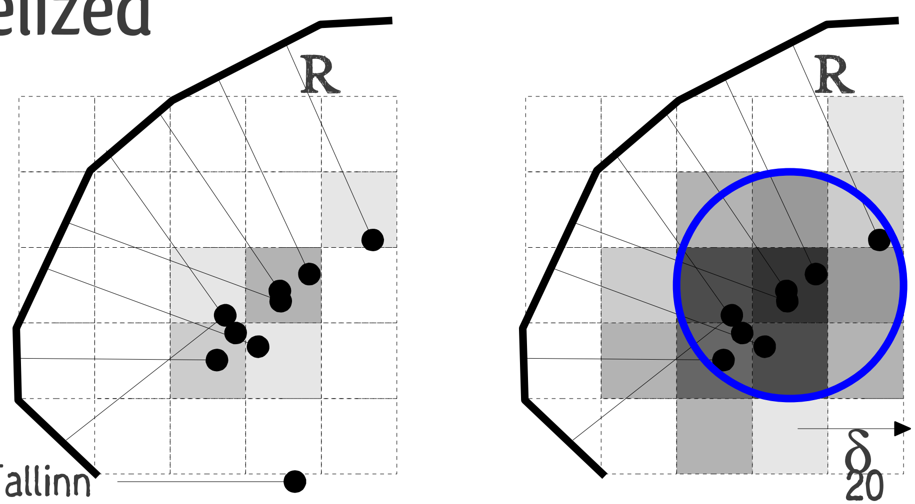


$$A' = A\,\vec{n}_0 \cdot \vec{n}(R_1)$$

Implementation(s)

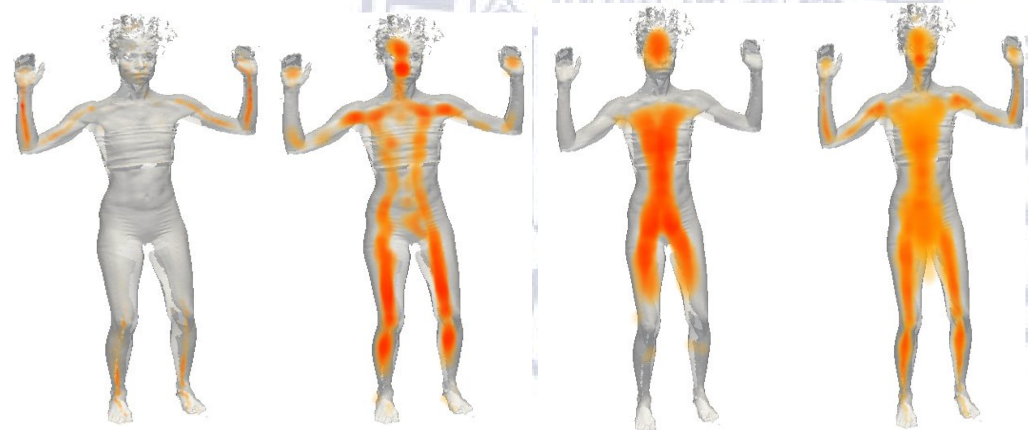
- Sample N points on triangles with approximately constant density. Assign to them area S(T)/N
- KD-Tree Method: store points in a KD-Tree structure and compute APT(**x**,R) summing contributions of points with distance < R from **x**
- Counting method: define a voxelized grid, sum contributions on grid then perform spherical (or arbitrary) kenel convolution





Results

- Visually good enhancement of centers of spheres and tubular parts
- Spatial behavior is robust against holes and noise
- Right: APT at different scales and JMAPT.
  - Scales sampled linearly



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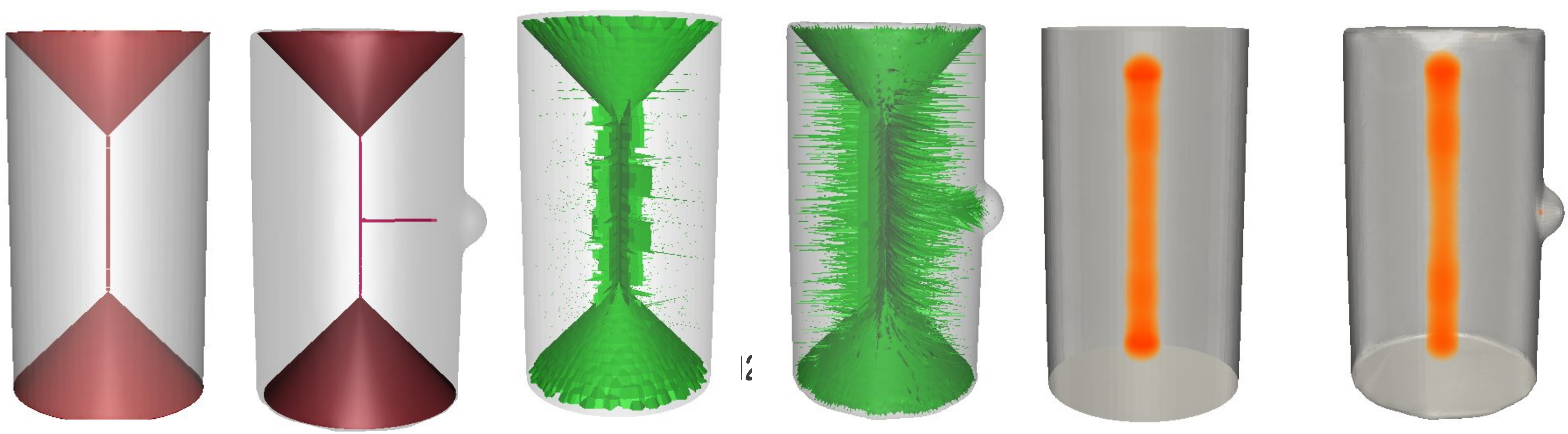
APT and Medial Axis Transform

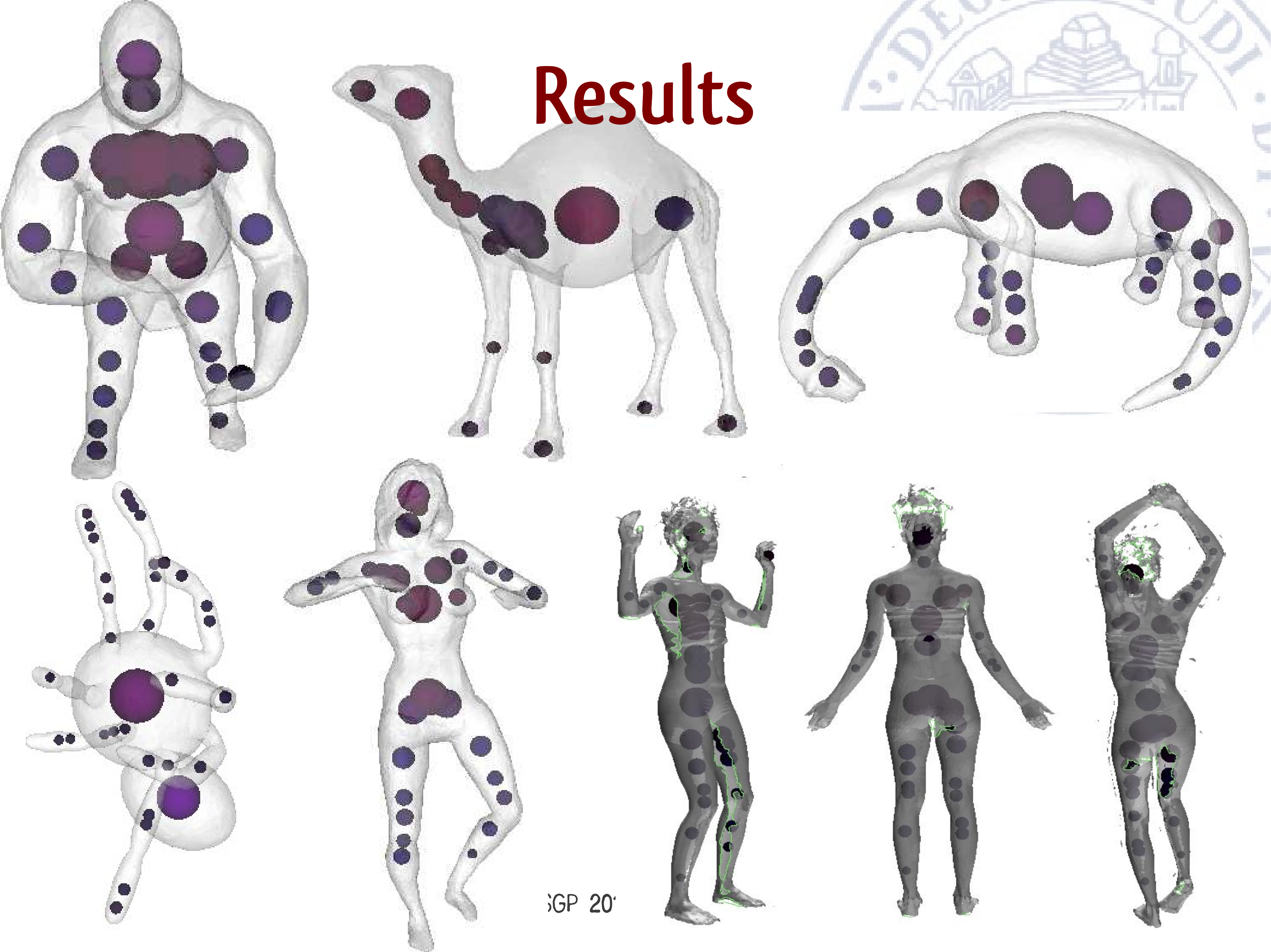
- The area density when $\to 0$ is nonzero in a subset of the medial axis
  - It is possible to obtain the complete medial axis with different definitions
- APT can be considered a robust volumetric medial axis
  - But not only scale selective like robust variant of medial axes (e.g Chazal et al. '05, Miklos et al. '10), but also made robust by area weighting



Applications

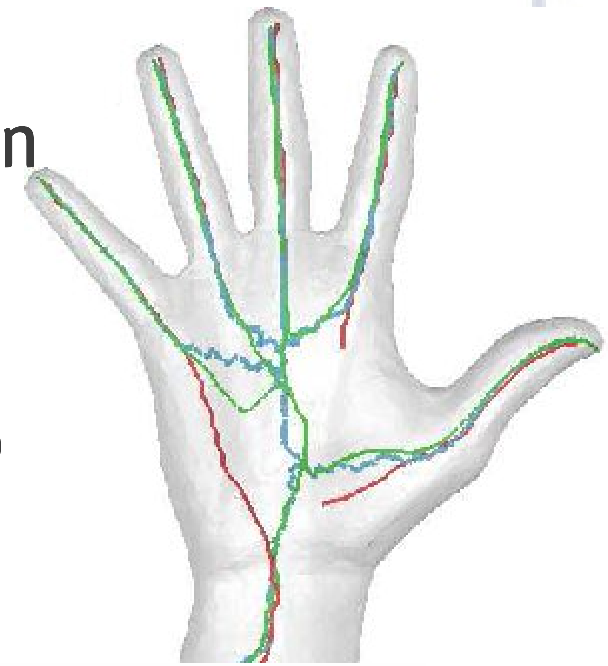
- Detection of salient points
  - Compute maxima of MAPT (JMAPT), possibly removing those under a fixed threshold or
  - Characterize them with a feature vector (scale, intensity, but also spatial behavior, e.g. eigenvalues of Hessian in a neighborhood)
- Detection of centerline of approximately tubular parts
  - Segment connected regions with high symmetry
  - Perform a vesselness enhancement and compute shortest path from highest values to fathest points
  - A more advenced skeletonization method based on APT is under development

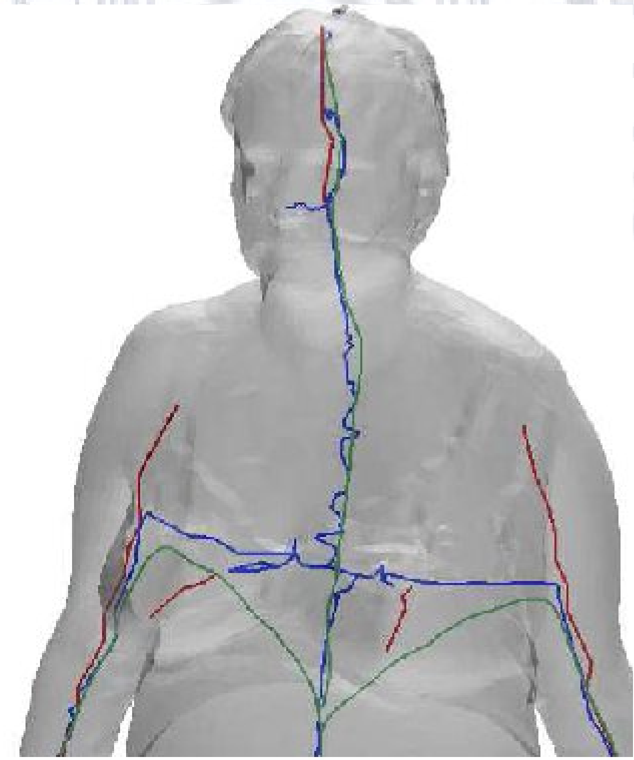
Results

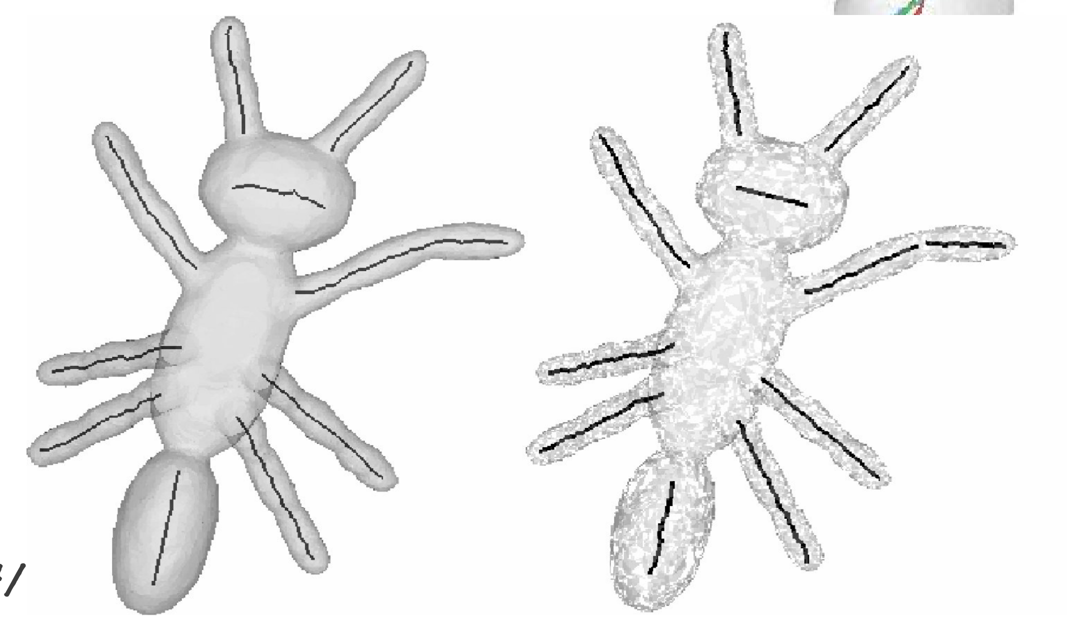


Results

- Skeletal lines extracted with JMAPT vesselness in red
- Lines follow cylindrical lines even if attached to other parts

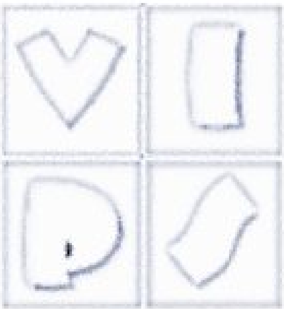






- Robust against holes (and noise)

Results



Shape retrieval

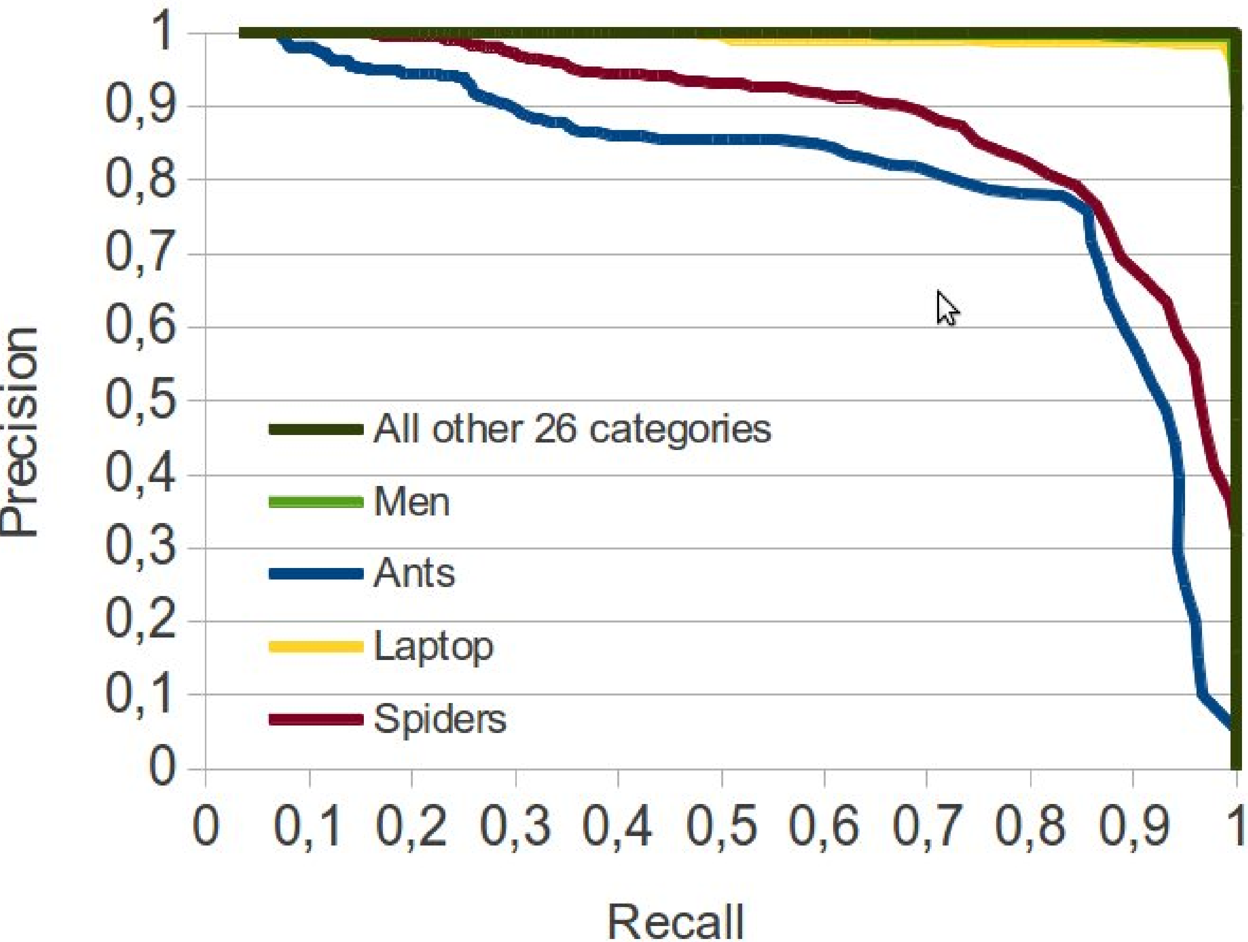
- We can characterize shapes using MAPT histograms (inside shapes)
- Appealing features:
  - Captures relevant features of natural shapes
  - Meaningful also for flat and non symmetric parts if kernel size is sufficiently large
  - Can be made perfectly scale invariant
  - Should perform well also in case of articulated deformations (only the region around bending points is changed.
- We created a simple MAPT based descriptor and tested it on the SHREC'11 nonrigid watertight database

HAPT descriptor

- Sample MAPT with 8 different R, proportional to the cubic root of object volume (square root of surface)
- Histogram sampled with 12 bins and concatenated, computed inside the shape

SHREC 2011 nonrigid testbed

- 30 categories 20 different articulated deformations
- NN classification with Jeffrey divergence
- Best than all the method in the contest
- Confused only by ants/spiders



	NN	1-Tier	2-Tier	e-Meas.	DCG
HAPT	**1.000**	**0.986**	**0.993**	**0.739**	**0.997**
SDGDM MeshSIFT	1.000	0.972	0.990	0.736	0.996
ShapeDNA	0.992	0.915	0.957	0.705	0.978
MDS-CM-BOF	0.995	0.913	0.969	0.717	0.982
MLSF	0.987	0.809	0.879	0.643	0.948
FOG+MRR	0.960	0.881	0.946	0.696	0.959
HKS	0.837	0.406	0.497	0.353	0.730

Discussion/Future work

- MAPT seems a powerful framework to extract salient points, tubular parts. A lot of possible future work on it
  - Matching relevant salient points (e.g. For anthropometric-biometric applications using point features and graphs)
  - Detection of small spherical bumps, crease lines, etc.
  - Development of a reliable curve skeleton method (selection of optimal scales, creation and cleaning of a set of paths, see)
  - Segmentation
- Histograms of APT seem a very good shape descriptor
  - Improvements using spatial information
  - Tests on real world classification tasks (e.g. shape morphometry in medicine)

Thank you

- Info: www.andreagiachetti.it/apt (soon available...)
- Post-doc position available (andrea.giachetti@univr.it)